AD-A266 635

RL-TR-92-300 Final Technical Report December 1992



VOLUME INTEGRAL EQUATIONS APPLIED TO CIRCULAR AND SQUARE CYLINDERS

University of Lowell Research Foundation

Rose W. Wang



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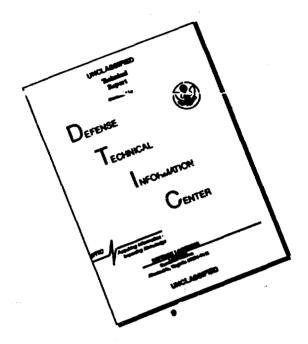


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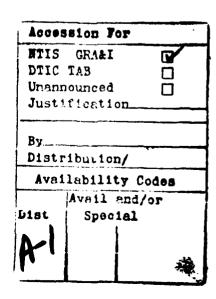
14. SUBJECT TERMS	SUBJECT TERMS Volume integral equations, Cylinders, Green's functions				
Volume integral equation		18 PRICE CODE			
	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION	20. LIMITATION OF ABSTRACT		
OF REPORT UNCLASSIFIED	OF THIS PAGE UNCLASSIFIED	OF ABSTRACT UNCLASSIFIED	U/L		

NSN 7540-01-280-5500

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1. INTRODUCTION

The method of moments (MOM) applied to volume integral equations has been a common way of solving for the electromagnetic scattering from highly inhomogeneous bodies. Many researchers [1] have experienced difficulties in obtaining an accurate solution for bodies with a high relative permittivity. With a revised treatment for the singular dyadic Green's function [2] in the integrand of the volume integral equation, we hope to overcome these difficulties and obtain a benchmark solution for a three-dimensional multi-wavelength scatterer with edges and corners.

Richmond [3],[4], and Livesay and Chen [5] appear to be the first to use the volume integral equation for dielectric bodies. Their procedure is based on volume discretizations of the volume integral equation with pulse basis functions and point matching. However, for 3-D problems as well as for the 2-D problems of TE polarization, substantial inaccuracies are observed. Recently, Joachimowicz and Pichot [6] compared the different integral formulations for 2-D TE scattering problems and analysed the source of these errors. They introduced a new integral equation that included

surface integrals to take account of surface charges at discontinuities. Even though better performance was obtained, considerable errors can still be observed.

In this report we are trying to accomplish two goals: (1) to eliminate the source of errors and (2) to obtain a benchmark solution to permit comparisons with the theoretically exact series solution for 2-D scattering from dielectric cylinders; and if successful, to obtain a benchmark solution for the 3-D scattering from a dielectric cube. Initial progress towards these goals are described.

At first, we believed the inaccuracies of the solution of the past researchers were probably due to inaccuracies in computing the highly singular dyadic Green's function for the scattering from dielectric cylinder, and in particular, in their treatment of the self term. In both Richmond and Chen's papers [3]-[5], the kernel of the integral equation is calculated by replacing the square self cell by a circular disk having the same area or replacing the volume self cell by a sphere having the same volume. Thus, our first step was to perform the self cell integration as accurately as possible. For the case of scattering from a

dielectric square cylinder, our results indicated that there is less than 1% difference between Richmond using the equal area approximation and our more accurate revised approach. Thus, there appears no great advantage in using a more accurate self cell evaluation in computing the bistatic scattering from the dielectric square cylinders.

In the original pulse basis and point matching method, the electric field and the dielectric properties are assumed to be constant in each cell and the Green's function is evaluated at the center of each cell. In other words, there is one center point for each nonself cell calculation. second approach, we increased the number of integration points of the Green's function for each of these cells and checked for convergence. For TM polarization, our results converged rapidly, and there is less than 2% difference between computing each cell contribution of the Green's function with one center point and with 240 integration points. However, our results converged more slowly for the TE case, the difference between computing with one center point, 25 points and 100 points, as compared with 240 points, was 5%,2% and 1% respectively. Again

the increase in accuracy was not very large.

Our next approach involved performing the integration over both the self and nonself cells by changing the area integrals to line integrals. Making use of Gaussian Legendre quadrature integration techniques, we can significantly decrease the total number of integration points from the number we used for the area integration and achieve higher accuracy. Our results suggest that the line integration method proved to be most accurate method for evaluating the Green's function over each cell. The area integration method required more than 200 integration points to achieve the same accuracy as 80 integration points with the line integration method.

One of the criteria for the pulse basis functions to work is to have cells much smaller than a wavelength. Since there is no exact series solution for the square cylinder, the circular cylinder was used for determining the required cell size. The analytical series solution for scattering from the circular cylinder can be easily derived and coded numerically. We first check the validity of analytical formulation of the circular cylinder and accuracy by comparing it with the results which can

be found in many electromagnetic textbooks [7] and [8]. The internal \overline{E} and \overline{H} fields for both TM and TE cases were computed and inspected to detect any unusual rapid changes in the fields. With the help of our exact code for scattering from the circular cylinders, the results with different cell sizes in the integral equations can be compared. The numerical results indicate that a minimal cell size of x/10 (where x is the wavelength in the dielectric material) in the TM case, gives approximately 2% of error in the far-field scattering from a long circular cylinder of radius = 0.3 x and permittivity equal to 3. Similar accuracy is observed in the TE case for low values of permittivity.

As mentioned above, three different approaches have been used to evaluate the Green's function over each cell - center point, area integration, and line integration, for the scattering from dielectric cylinders. There is practically no distinction between the three methods in computing scattering from the dielectric cylinders in the case of TM polarization. The results are in good agreement with the exact solution to less than 5% maximum error. However, this is not the case

for TE polarization, when the permittivity is large or when the size of the circular cylinder is large. Even though the cell size decreases, better agreement is needed between the numerical data and exact solution. The curves tend to converge very slowly to the exact curve.

One explanation of the deficiency may be due to the square cell representation of the circular cylinder. The jagged edges may produce an error. Tables 1 and 2 show the comparsion between different cells size for the scattering from a dielectric circular and square cylinder for $\epsilon r = 3$, radius = 0.3λ (k_a = 1.88), for TM and TE polarization respectively. Observed that, unlike the square cylinder case, the results for the circular cylinder oscillate about the exact solution as the cell size decreases or the number of cells per dielectric wavelength increases. However, the oscillations are quite small. This suggests that the jagged edges cause minor oscillations, and are not the main source of the error. We tried to verify this by computing the RCS versus koa of the square and circular cylinders with large permittivity ($\epsilon r = 10$), where ko is the free space wave number, and 'a' is

		Circular Cylinder		Square Cylinder er-3, k _o a-1.88	
N	٨/۵ *	RCS 0 (dB)	RCS 180 (dB)	RCS · 0	RCS 180 (dB)
5 6 7 8 9 10 11 12 13	10 12 13 15 17 19 21 23 25 27	0.39695 0.48410 0.29716 0.32672 0.41695 0.44676 0.43276 0.48777 0.38840 0.44939	10:19377 9.94038 10 38816 1(.34977 10:16945 10:07070 10:09018 9.97323 10:21299 10:10223	-2.16554 -2.21087 -2.23595 -2.25133 -2.26145 -2.26848 -2.27357 -2.27737 -2.28030 -2.28259	11.36273 11.37733 11.38627 11.39211 11.39614 11.39903 11.40117 11.40280 11.40406 11.40507
15 Exact	29 Soln.	0.41067 0.48567	10.15351	-2.28443	11.40588

Table 1. Comparison between Bistatic scattering of Circular and Square dielectric Cylinders, TM polarization using different size of square cells.

^{*} x dielectric wavelength

Δ size of square cell

N number of segments per radius.

		Circular Cylinder er= 3, k _o a = 1.88		Square Cylinder er= 3, k _o a = 1.88	
N	λ, Δ *	RCS 0 (dB)	RCS 180 (dB)	RCS 0 (dB)	RCS 180 (dB)
5 6 7 8 9 10 11 12 13 14 15	10 12 13 15 17 19 21 23 25 27 29	-11.24765 -11.03964 -12.36419 -13.65737 -12.52884 -12.61856 -13.06285 -12.26397 -13.66110 -12.82131 -13.62015	8.97380 8.71287 8.93033 9.12377 8.86450 8.86981 8.90101 8.71944 8.97206 8.97206 8.93140	-11.40775 -11.37207 -11.33195 -11.29794 -11.27067 -11.24895 -11.23151 -11.21733 -11.20564 -11.19589 -11.19541	10.93251 10.92116 10.91392 10.90898 10.90544 10.90280 10.90077 10.89916 10.89785 10.89678 10.89599

Table 2. Comparison between Bistatic scattering of Circular and Square dielectric Cylinders, TE polarization using different size of square cells.

^{*} x dielectric wavelength

Δ size of square cell N number of segments per radius.

the radius of the circular cylinder or half-side length of the square cylinder. The curves for different cell sizes agree quite closely up to a cylinder size of koa equal to about 1.5 for both the square and circular cylinder (see Fig. 70). Especially notice that the scattering curves of the dielectric square cylinder strongly separate as koa increases beyond 3.0. This result again indicates that the jagged edge in the circular cylinder case is not the main source of the error.

Despite our improved computational accuracy, the method of moments with pulse basis functions and point matching for bistatic TE scattering from large cylinders with large permittivities did not give accurate results. Some authors have suggested that the volume integral equation solved with pulse basis functions produces a false surface charge density [1]. Using pulse basis functions to represent the unknown polarization introduces fictitious charge layers at every cell boundary. Nevertheless, some researchers have been successful in obtaining accurate results with more sophisicated techniques, such as linear basis functions with polygonal cells [9], polyhedral cells [10], rooftops basis functions with triangular cells [11], square cells

[12], and tetrahedral cells [13]. Still, a serious drawback of these methods is the large number of unknowns required per cell.

Since the existing volume integral equation solved with pulse basis functions does not deal adequately with surface charge density in the TE polarization case, we are presently revising the formulation of the volume integral equation to retain pulse basis functions and yet avoid the fictitious charge density problem.

2. EXACT SOLUTION FOR THE CIRCULAR DIELECTRIC CYLINDER

Our first task is to generate a general computer code for the exact solution of bistatic scattering from a lossless dielectric circular cylinder (Fig. 1) The results of this code are then compared to the exact solutions found in many electromagnetics scattering books and papers. We select Ruck's Radar Cross Section Handbook [7] and Barber's Light Scattering by Particles book [8] as our reference for their more comprehensive examples.

With our exact code developed, the internal fields can also be obtained. This is done to see if there is any abnormality or large variation in magnitude and phase in the internal fields that requires special attention or treatment.

The exact solution is then compared to the MOM solutions using the center point, area and line integral approaches for evaluating the Green's function over the self cells to evaluate the bistatic scattering from an infinite circular lossless dielectric cylinder. Finally, we will apply the MOM codes to infinitely long square lossless dielectric cylinders.

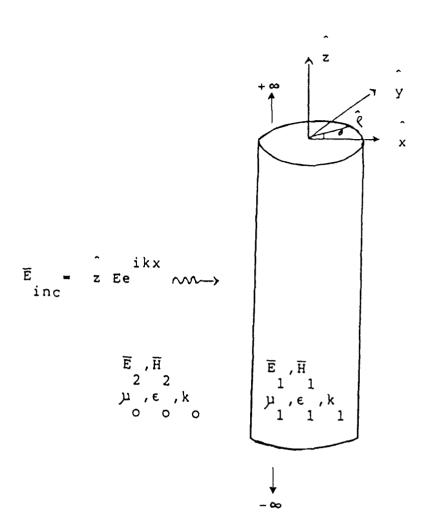


Fig. 1 Scattering from a dielectric cylinder

2.1 Brief Derivation of Exact Formula

2.1.1 TM Case

$$\begin{array}{cccc}
& & & & & & \\
\text{Let } E & = & z e & & & \\
\end{array} \tag{1}$$

$$\overline{E} = z E \qquad \qquad g \leq a \qquad (2)$$
1 1z

$$\overline{E} = \overline{E} + \overline{E}$$
 $f \ge a$ (3)
2 scat inc

From Maxwell's equation

$$\nabla X \overline{E} = iw\mu \overline{H}$$
 (4)

$$H_{\phi} = \frac{i}{\psi} \frac{\partial E}{\partial g}$$
 (5)

$$H_{g} = \frac{1}{i_{g} w_{jl}} \frac{\partial E}{\partial g}$$

$$(6)$$

Using boundary conditions for \overline{E} and \overline{H} at f = a tan tan

$$\begin{array}{ccc}
E & = & E \\
1z & 2z
\end{array} \tag{7}$$

$$H = H \tag{8}$$

$$1\phi \qquad 2\phi$$

and letting

$$E = \sum_{\substack{n=\infty \\ 12}}^{\infty} a J(k f) e \qquad \qquad f \leq a$$
(9)

$$E_{2z} = \left\{ \sum_{n=-\infty}^{\infty} t H(kg) + \sum_{n=-\infty}^{\infty} J(kg) \right\} e^{in\phi} \qquad \beta \geq a \qquad (10)$$

$$H_{1\phi} = \frac{i k_1 \sum_{n=-\infty}^{\infty} a}{w \mu_1^{n=-\infty} n} J'(k_{\beta}) e^{i n \phi}$$

$$\beta \leq a$$
(11)

$$H_{2\phi} = \frac{ik \sum_{n:-\infty}^{\infty} (1)}{\sum_{n:-\infty}^{\infty} (n - n)} + \sum_{n=-\infty}^{\infty} J'(k s) e \qquad \qquad s \ge a$$

$$\lim_{n\to\infty} (12)$$

we find

$$a = \frac{k \left\{ J(k, \beta) H'(k, \beta) - H(k, \beta) J'(k, \beta) \right\}}{n o' n o'}$$

$$k J(k, \beta) H'(k, \beta) - k H(k, \beta) J'(k, \beta)$$

$$o n 1 n o 1 n o' n 1$$

(13)

$$t_{n} = \frac{k \quad J(k g)J'(k g) - k J(k g)J'(k g)}{1 \quad n \quad o \quad n \quad 1 \quad n \quad o}$$

$$k \quad J(k g)H'(k g) - k H(k g)J'(k g)$$

$$o \quad n \quad 1 \quad n \quad o \quad n \quad 1$$

(14)

2.1.2 TE Case

$$\frac{\text{inc}}{H} = z e$$
(1)

$$\frac{\overline{H}}{2} = \frac{\overline{H}}{1} + \frac{\overline{H}}{1}$$
 $\rho \geq a$ (3)

From Maxwell's equation

$$\nabla X \overline{H} = -iw\varepsilon \overline{E}$$
 (4)

$$E_{q} = \frac{1}{iw\epsilon} \frac{\partial H}{\partial g}$$
 (5)

$$E_{g} = \frac{i}{w_{\varepsilon}} \frac{\partial H}{\partial g} . \tag{6}$$

Using boundary conditions for \overline{E} and \overline{H} at β = a tan tan

$$\begin{array}{ccc}
E & = & E \\
1\phi & 2\phi
\end{array} \tag{8}$$

and letting

$$E_{2\phi} = \frac{k_0 \sum_{n=\infty}^{\infty} s H'(k_g) + \sum_{n=\infty}^{\infty} J'(k_g)}{iwe^{n \log n}} + \sum_{n=\infty}^{\infty} J'(k_g)} e^{in\phi}$$
(12)

we find

$$\frac{k_{o}}{-\epsilon_{o}} \left\{ J_{(k_{o})H'(k_{o})} - H_{(k_{o})J'(k_{o})}^{(1)} \right\} \\
= \frac{k_{o}}{-\epsilon_{o}} I_{(k_{o})H'(k_{o})} - \frac{k_{i}}{-\epsilon_{o}} I_{(k_{o})J'(k_{o})}^{(1)} \right\} \\
= \frac{k_{o}}{-\epsilon_{o}} I_{(k_{o})H'(k_{o})} - \frac{k_{i}}{-\epsilon_{o}} I_{(k_{o})J'(k_{o})}^{(1)}$$

(13)

(14)

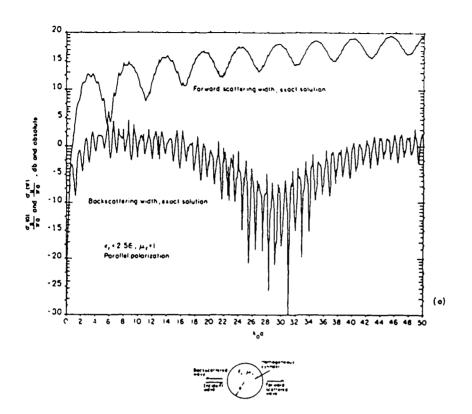
3. RESULTS FROM THE EXACT CODE

The results for the far field scattering obtained from the exact solution in Section 2.1 of the dielectric lossless circular cylinder are compared to the exact solutions from Ruck's Radar cross section Handbook [7], (Figs. 2-3) and Barber's Computational Methods [8] (Figs. 4-10).

Figs. 2-3 show the normalized scattering cross section of a circular cylinder with $\epsilon r = 2.56$, $\mu r = 1.0$ with varying k_oa for vertical polarization (TM) and perpendicular polarization (TE) respectively. Figs. 2a and 3a are the exact solutions from Ruck's book. Figs. 2b and 3b are from our simulation. The good agreement confirms our numerical code for the exact solution.

Figs. 4-5 show the angular scattered intensity for a circular cylinder with size parameter of 50 and an index of refraction of 1.5 for TM and TE polarization. The plotting increment is 0.2 degree. The solutions from Barber's book are shown in Figs. 4a and 5a respectively. Agreement with Barber's solution is good.

Figs. 6-7 show the internal intensity along the



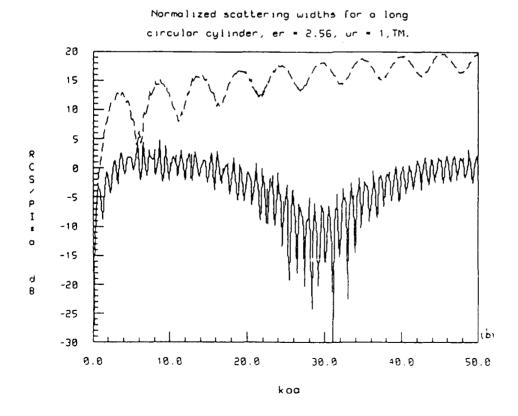
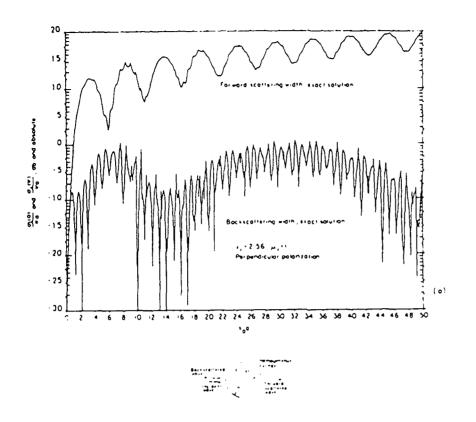


Fig. 2



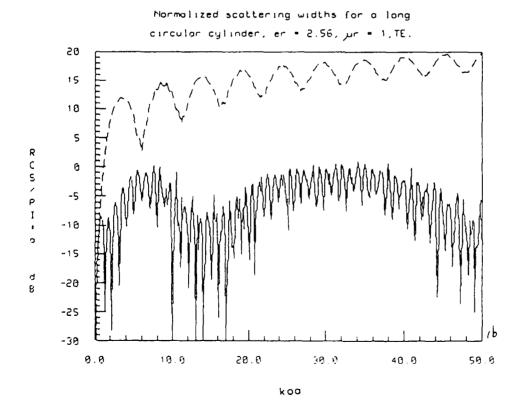
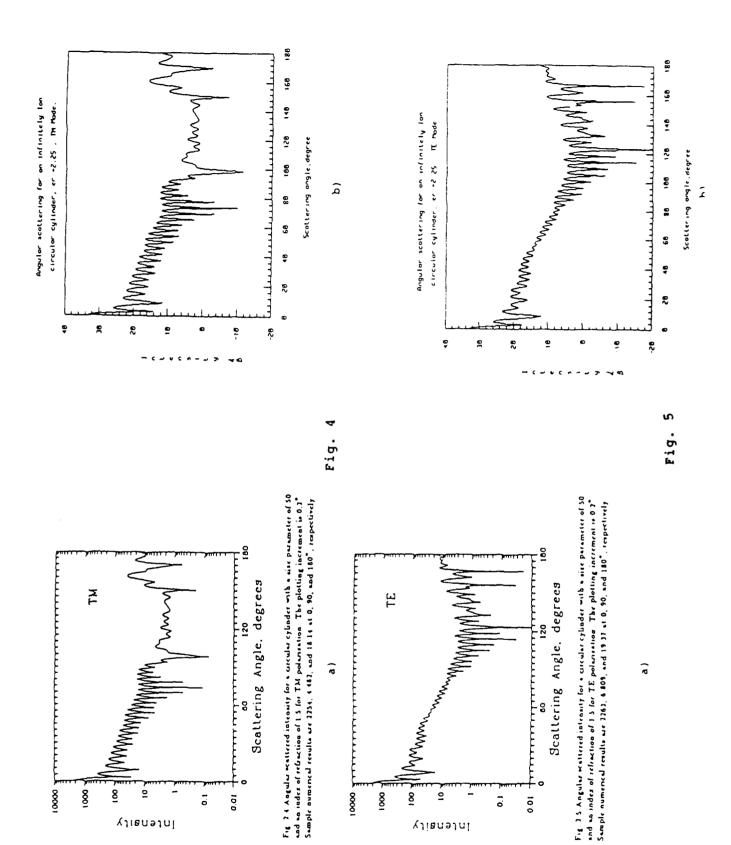
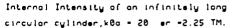
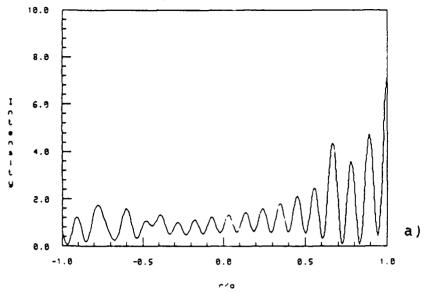


Fig. 3







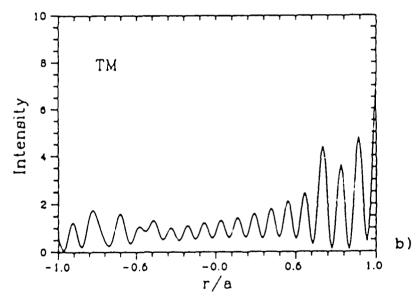


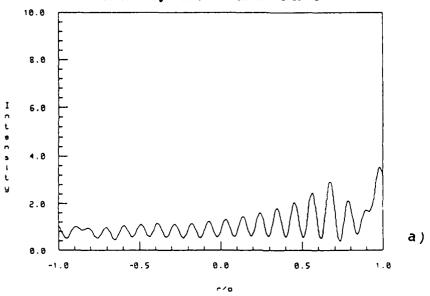
Fig. 2.28 The internal intensity along the a axis as a function of r/a for a circular cylinder with a size parameter of 20 and an index of refraction of 1.5 for Th ; olarisation. The incident wave propagates from left to right. The calculation used 400 points. Sample numerical results are 0.6810 and 7.569 at r/a=-1 and +1, respectively.

x-axis as a function of r/a for a circular cylinder with a size parameter of 20 and an index of refraction 1.5 for TM and TE polarization. The incident wave propagates from left to right. The calculation used 400 points. The exact computed solutions (Figs. 6a-7a) are in excellent agreement with the reference solutions (Figs. 6b-7b).

Figs. 8-10 show the scattering intensity at 0, 90 and 180 degrees as a function of size parameter for a circular cylinder with an index refraction of 1.5 for TM polarization. The calculations used 1001 points. Figs. 8a-10a are the computed solutions. The agreement with the reference solutions (Fig. 8b-10b) are excellent.

We can thus conclude that our exact solution code for the scattering from a long circular cylinder is working reliably.

Internal Intensity of an infinitely long circular cylinder,k8a = 28 en =2.25 TE.



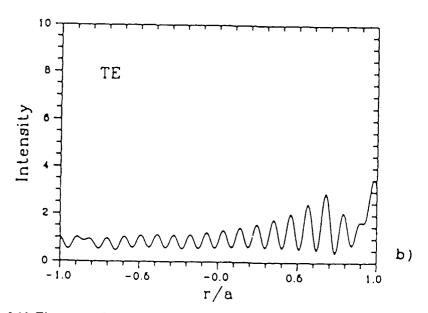
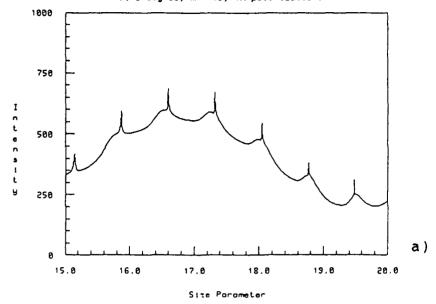


Fig. 2.30 The internal intensity along the z axis as a function of r/a for a circular cylinder with a size parameter of 20 and an index of refraction of 1.5 for TE polarization. The incident wave propagates from left to right. The calculation used 400 points. Sample numerical results are 1.055 and 3.131 at r/a=-1 and +1, respectively

Scattering Intensity for cir. cylinder at 0 degree, m =1.5, TM polarization.



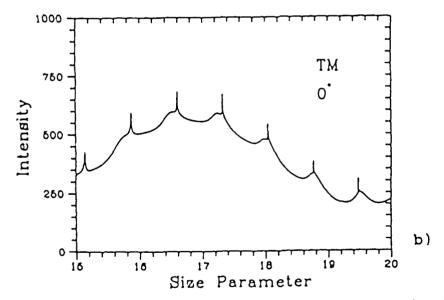
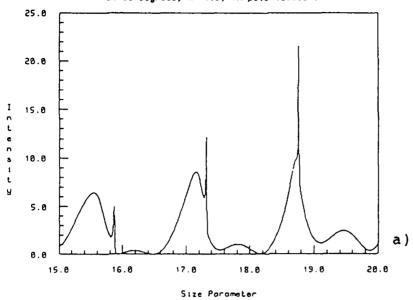
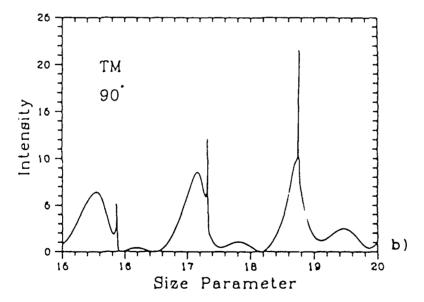


Fig. 2.12 Scattered intensity at 0° as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM polarization. The calculation used 1001 points. Sample numerical results are 328.1, 512.5, and 222.3 at size parameters of 15, 17.5, and 20, respectively.

Fig. 8

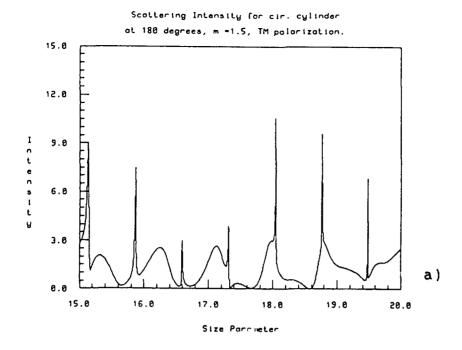
Scattering Intensity for cir. cylinder at 90 degrees, m =1.5, TM polarization.





Pig. 2.13 Scattered intensity at 90° as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM polarisation. The calculation used 1001 points. Sample numerical results are 0.7907, 0.4846, and 1.073 at size parameters of 15, 17.5, and 20, respectively.

Fig. 9



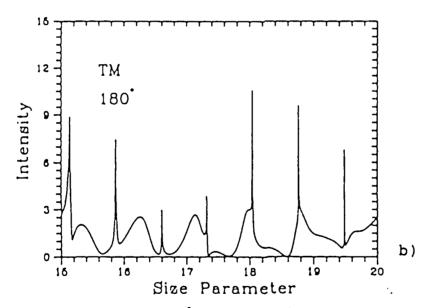


Fig. 2.14 Scattered intensity at 180° as a function of size parameter for a circular cylinder with an index of refraction of 1.5 for TM polarisation. The calculation used 1001 points. Sample numerical results are 2.725, 0.3091, and 2.519 at size parameters of 15, 17.5, and 20, respectively.

4. INTERNAL FIELDS

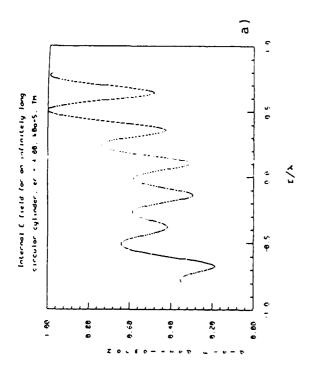
The Internal fields of the dielectric circular cylinder can be computed using the equations of Section 2.1 for both the TM and TE cases.

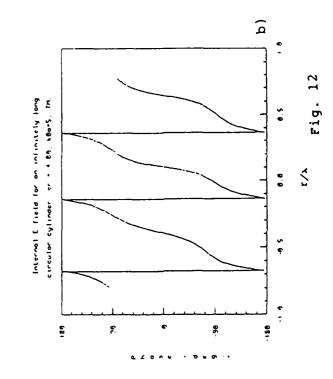
The magnitudes of the E fields across the diameter with various relative permittivities of an infinitely long circular dielectric cylinder are shown in Figs. (11-15)a and the phases in Figs.(11-15)b. The H fields cases (TE polarization) are shown in Figs. 16-20.

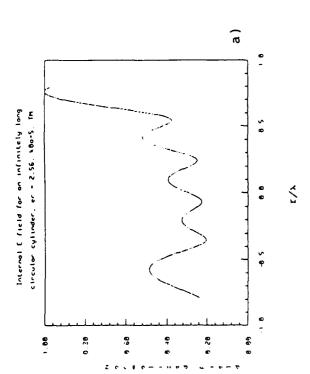
Fig. 11 shows the normalized internal E field (TM case) versus distance (r/x) across the diameter of the cylinder with size parameter $k_oa = 5$ and relative permittivity of 2.56. The calculation used 400 points in r/x. The normalized internal field is defined as |E|/|Emax| or |H|/|Hmax|.

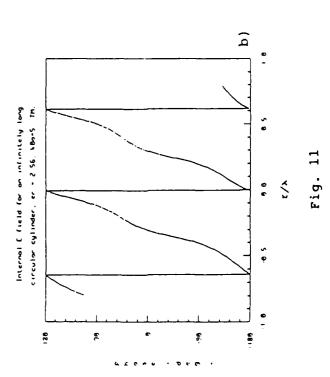
Figs. 12-15 show the normalized internal E field versus distance (r/λ) across the cylinder $(k_0a = 5)$ with relative permittivities = 4,10,20 and 50, respectively. Similarly, the H fields from TE polarization are shown in Figs. 16-20.

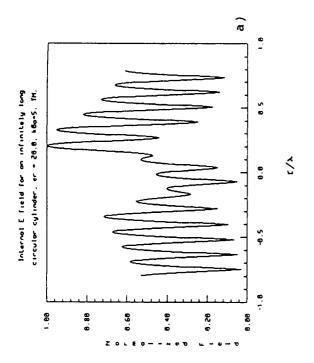
Figs. 21-30 show the normalized internal field

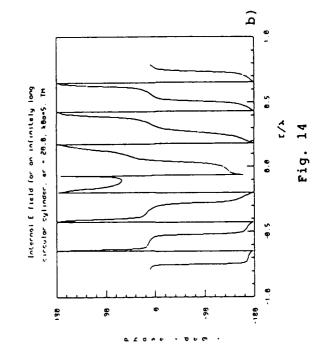


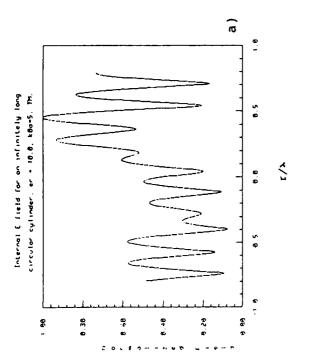


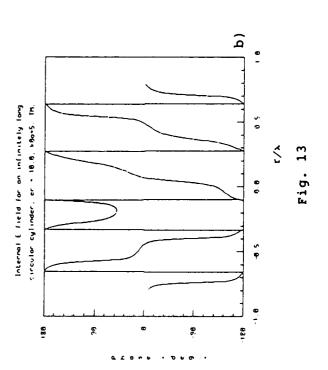


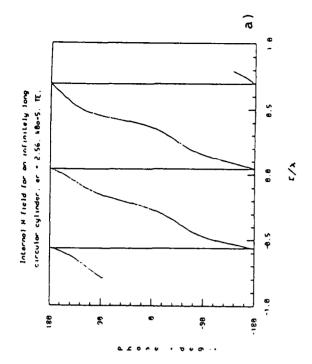


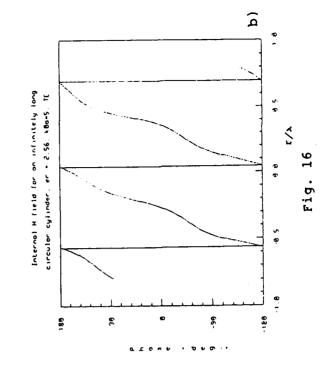


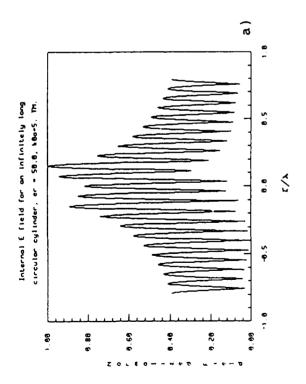


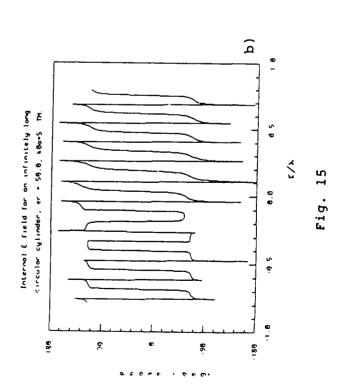


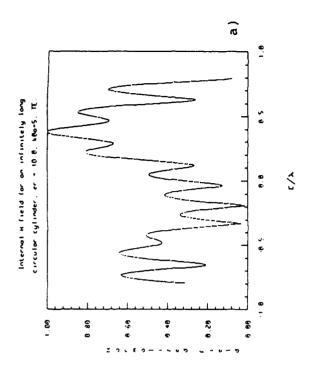


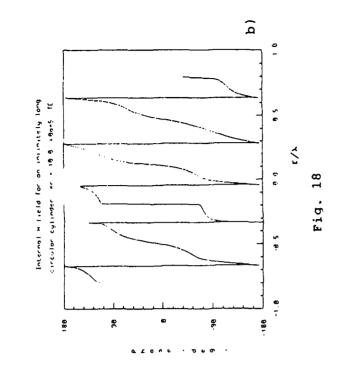


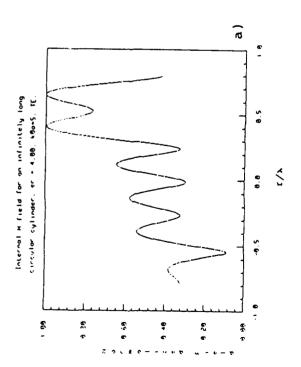


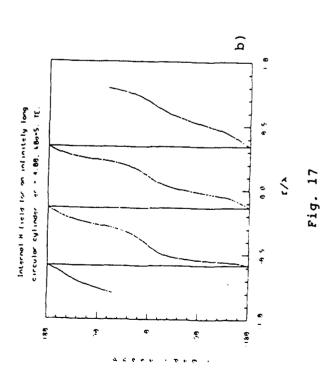


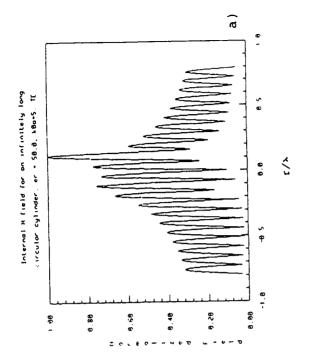


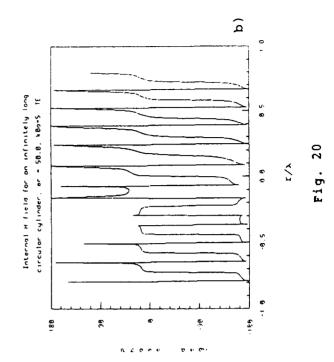


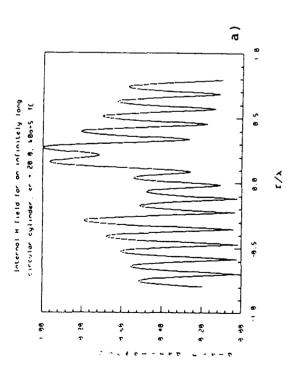


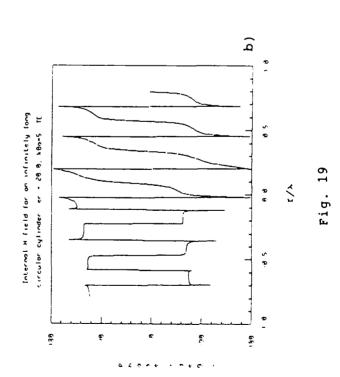












with various angles (0,45,90,135,180 degrees) and relative permittivities (2.56,4,10,20,50) as a function of distance across the radius of the cylinder for both polarizations. It can be seen that the internal field at 0 degrees when combined with the internal field at 180 degrees give the internal field across the dielectric circular cylinder (see Figs. 11-20).

The purpose of finding the internal fields of the dielectric lossless cylinder with different scattering angles and various permittivities is to see if the internal field varies unexpectedly rapidly near the surface of the large dielectric lossless cylinders. If this were the case, special treatment near the discontinuity or along the edges of the cylinder would be necessary, that is, more cell divisions would be required in our numerical solution. A standing wave pattern can be observed from the figures. Nevertheless, there is no sudden jump nor localization of the internal field near the rim of the large dielectric lossless cylinder. Our results indicate that even though the relative permittivity is large ($\epsilon r=50$), the size of the cylinder is still small (k, a=5) enough that the internal fields did not accumulate near

1.88 •06 135 180 45 0.80 Internal E field for an infinitely long circular cylinder, er = 2.56, $k\theta\alpha$ =5. TM. 0.60 0.40 0.50 8.88 8.88 B.8B 0.60 0.20 0.40 1.00 Zoleo-- N @ 7 σ - φ

Fig. 21

ر ک

1.00 180 0.80 Internal E field for an infinitely long = 4.88, k8a=5. TM. 09.0 circular cylinder, er 0.40 0.20 00.00 08.0 09.0 0.40 0.20 0.00 1.00 Z 0 L E 0 - - N 0 0

Fig. 22

?

Internal E field for an infinitely long circular cylinder, er = 10.0, k0a=5. TM.

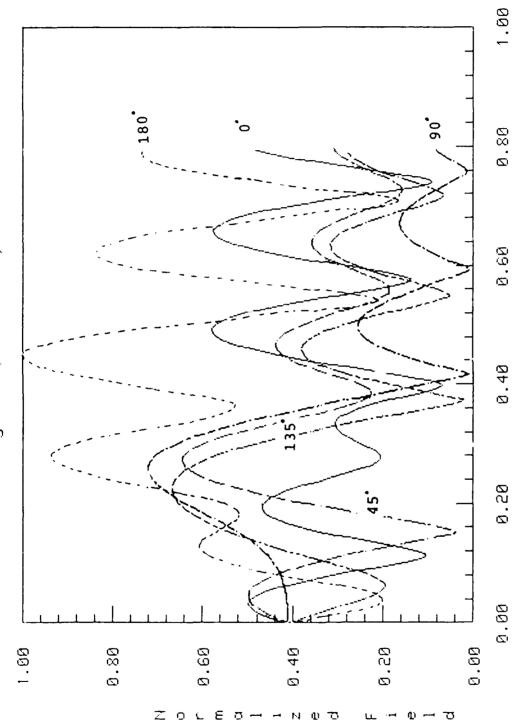
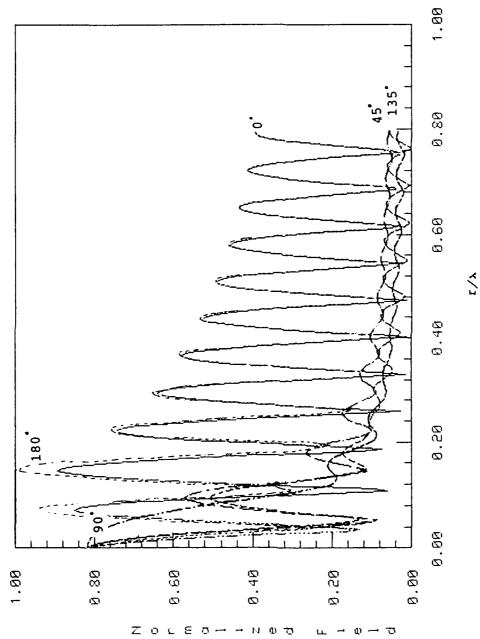


Fig. 23

0.80 Internal E field for an infinitely long circular cylinder, er = 20.0, kBa=5. TM. Ø.60 0.40 0.20 0.00 1.88 0.39 0.68 9.20

Fig. 24

Internal E field for an infinitely long = 50.0, k0a=5. TM. circular cylinder, er



1.00 45, 08.8 Internal H field for an infinitely long = 2.56, k0a=5. TE. 0.60 circular cylinder, er 0.40 0.50 Ø.08 08.80 1.00 0.60 0.40 0.20 0.00 Ζοιεσ-υ — σ

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39

1.00 •06 0.80 Internal H field for an infinitely long circular cylinder, er = 4.00, k0a=5. TE. 09.0 ۲ ک 0.40 0.20 0.00 0.00 1.00 8.89 0.68 0.20 C E 2 ~

Fig. 27

1.00 08.80 Internal H field for an infinitely long circular cylinder, er = 10.0, k0a=5. TE. 0.68 0.40 0.20 0.00 1.00 08.0 0.60 0.20 0.40 0.00 2 ع

Fig. 28

1.88 0.80 Internal H field for an infinitely long 20.0, k0a=5. 0.68 circular cylinder, er 0.40 0.50 1.00 0.80 0.50 00.00 0.40 0.60

Fig. 29

Internal H field for an infinitely long circular cylinder, er = 50.0, k0a=5. TE.

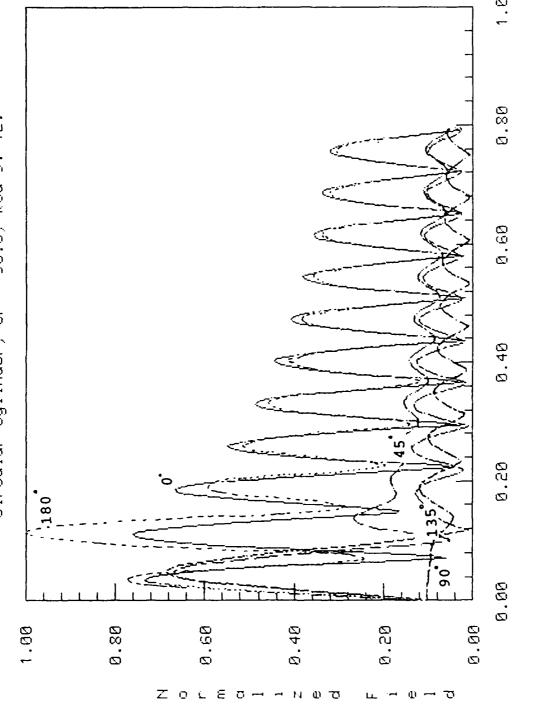


Fig. 30

the surface. (However, for large cylinders (k_oa = 45), the internal fields do localize near the surface [8].) Furthermore, the total number of variations of the internal fields across the cylinder is consistent with the value of the relative permittivity. Since there is no abnormality in the internal fields for the cylinder size and dielectric constants we considered, no special treatment was needed in the numerical solution to take this into account.

5. COMPARISON BETWEEN EXACT SOLUTIONS AND THE MOM NUMERICAL SOLUTIONS USING THE AREA AND LINE INTEGRATION METHODS

The derivation of the area integration method can be found in Marian Silberstein's report, Electromagnetic Scattering from Dielectrics - A Two-Dimensional Integral Equation Solution [14]. This approach uses the method of moments (MOM) with pulse basis functions and point matching to discretize the integral equation into a linear system of equations which is then solved by matrix inversion. That is, the electric field was represented by a constant function within each cell and the equation was enforced at the cell centers. To ensure good accuracy, the size of each cell must be small compared to the dielectric wavelength, (i.e., the number of cells n per dielectric wavelength λ , should be > 10). For the circular cylinder, the results are compared to the exact series solution and are shown in Figs. 31-40.

Figs. 31-37 show the normalized bistatic TE scattering from a long circular dielectric cylinder with different relative permittivities and values of $k_{o}a$. The number of cells per dielectric wavelength is set to 10 and the center point integration method is used. Excellent agreement

180 exact ---- MOM _ 전 circular cylinder, koa=1.0, er=1.05,TE. Normalized Scattering for a long ref: Radar Cross Section Handbook (Ruck) 120 (Z) <u>60</u> 98 Ø -120 30 -30 -60 -90

 $\underline{\omega} \in \mathbb{C}^{n}(\omega)$

Fig. 31

angle (degrees)

ಶಹ

circular cylinder, er=2.56, koa=0.7 TE. Normalized Scattering for a long

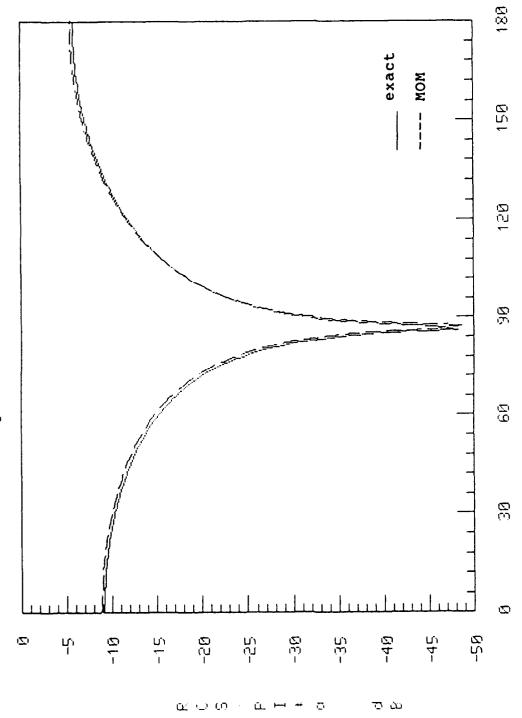


Fig. 32

angle (degrees)

exact MOM ---circular cylinder, er=9, koa=1.0 TE. Normalized Scattering for a long 0 0 20 Ť. (2) ďί -10 <u> 1</u> -2<u>A</u> -25 ហ Ď

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Fig. 33

angle (degrees)

188

150

120

(A)

<u>6</u>0

90

 $\overline{\omega}$

က ယ

Normalized Scattering for a long circular cylinder, er=9.5, koa=.7 TE.

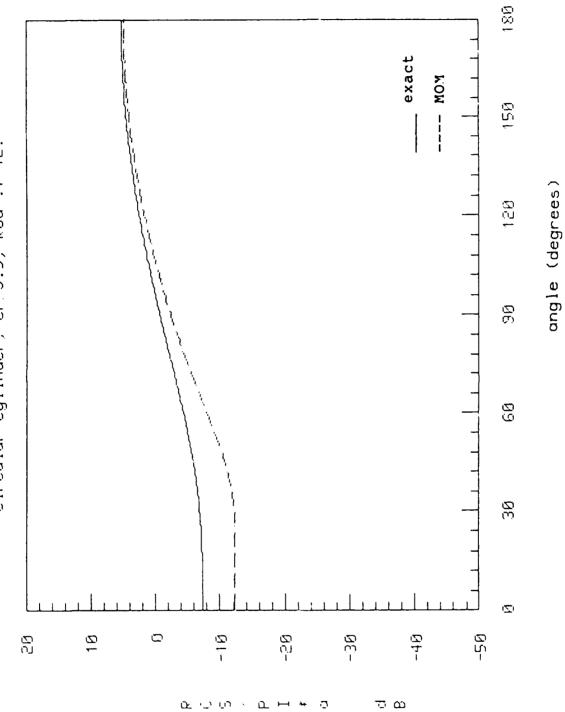


Fig. 34

is observed with the exact series solution when the relative permittivity or size of the cylinder is small (see Figs. 31-33 and Figs. 35-37).

Figs. 34 shows the TE bistatic scattering from a circular cylinder versus scattering angles for $k_0 = 0.7$ and a large relative permittivity of 9.5. As seen from the figure, the MOM solution tends to deviate from the exact solution for large relative permittivity.

For TM polarization, the normalized bistatic scattering from a long dielectric circular cylinder is shown in the Figs. 38-40. Fig. 38 shows the normalized bistatic scattering cross section from a long circular dielectric cylinder with $\epsilon r = 2.56$ and $k_c a = 0.7$. Figs. 39-40 show that even with large relative permittivities, the agreement between the computed solution and the reference solution is good.

The following figures show the comparison between the exact solution and the integral equation solution using the center point integration method for a circular cylinder with varying k_0a . Relative permittivities of 2.56,4, and 10 are chosen. Both TM and TE polarization

180 exact MOM ----15<u>8</u> circular cylinder, a=4.25cm, er=1.001,TE Normalized Scattering for a long 128 angle (degrees) (四) (7) 60 30 Ø -120 30 -30 $\overline{\Omega} \psi^+$ $\alpha \cup \omega$ ಶಹ

51

180 exact MOM ----158 circular cylinder, a=4.25cm, er=1.010,TE Normalized Scattering for a long 120 angle (degrees) 96 99 30 $\overline{\Sigma}$ 30 Ø -120 -38 168 -96 α \cup α \wedge ಶಜ

Fig. 36

2 0 0 exact MOM ----150 circular cylinder, a=4.25cm, er=1.100,TE Normalized Scattering for a long 120 98 68 38 Ø (C) ıΣı -38 -128 -68 -98 $\alpha \cup \phi$ ထင

Fig. 37

angle (degrees)

a --- --

180 160 exact ---- MOM circular cylinder, er =2.56, koa=.7, TM 140 Normalized Scattering for a long 120 angle (degrees) 100 80 68 40 20 Ø -30 20 18 Ø -10 -20 **8** 9 ж U S / В H * ۵

54

180 168 exact MOM ----140 circular cylinder, er =9.5, koa=.7, TM Normalized Scattering for a long 128 angle (degrees) 100 80 69 48 20 Ø -38 20 10 Ø -18 -20 ထတ A O S / A H

Fig. 39

180 exact 160 ---- MOM 140 circular cylinder, er =50, koa=.7, TM Normalized Scattering for a long 120 100 80 9 40 20 Ø Ø -30 20 10 -18 -20 8 U S / L H * B <u>σ</u> ω

angle (degrees)

are considered. We used three different methods of approximation for evaluating the self cell terms to compare their efficiency. First, the self cell term is approximated by using a constant area with one center point. For case 2, the number of integration points are increased to 25 points using the same approximation method. Thirdly, the self cell term is evaluated by using the line integral method with 80 points.

Figs. 41-43 show the forward scattering from a circular cylinder versus koa for relative permittivity = 2.56,4 and 10, respectively, for the TM polarization, the solid curve represents the exact solutions. The exact solution uses 1200 k,a points. In our numerical computations, the number of cells per dielectric wavelength is set to 10. to limitations of computer time, only 120 k, a points are used in our numerical evaluation. Figs. 44-46 show the back scattering. Good agreement is obtained between the three different numerical methods and the exact solution. However, for $\epsilon r=10$, better agreement can be attained if more k_0 a points (N > 120) and the number of cells (n/x) per dielectric wavelength are increased in the computation.

5.00 exact MOM ----4.00 Forward Scattering for a long circular cylinder, er = 2.56, n/x = 10, TM 3.00 2.00 1.88 0.00 -38 20 Ø -20 10 -10 æσ α \circ \circ \circ \circ \circ \circ ۵

Fig. 41

koα

3.00 exact MOM ----2.50 Forward Scattering for a long circular cylinder, er = 4.80, n/x = 10, TM 2.00 1.50 1.88 0.50 0.00 -38 20 10 Ø -10 -20 & O S / P H * മെ ۵

Fig. 42

koa

2.00 exact ---- MOM Forward Scattering for a long circular 1.50 cylinder, er = 18.8, n/x = 18, TM koa 1.00 0.50 0.00 -30 Ø -10 -20 20 10 **& U S / P H *** B B

Fig. 43

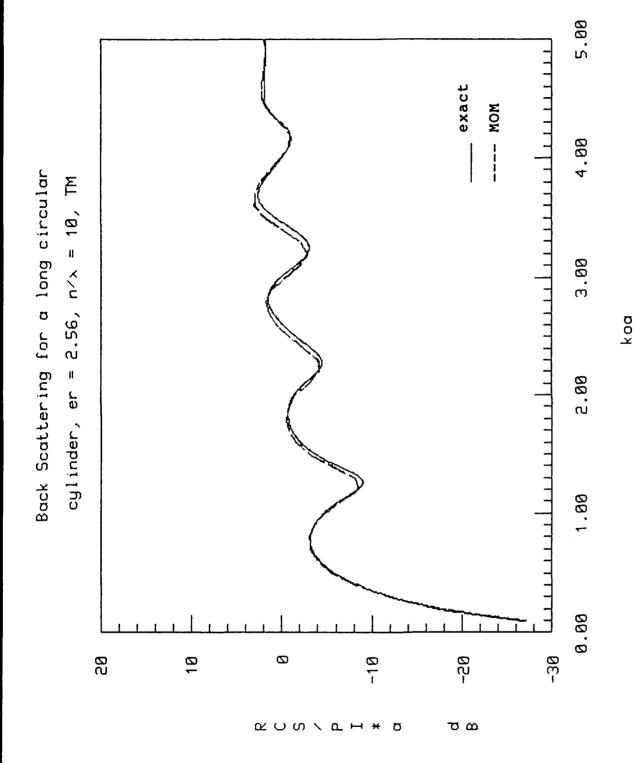


Fig. 44

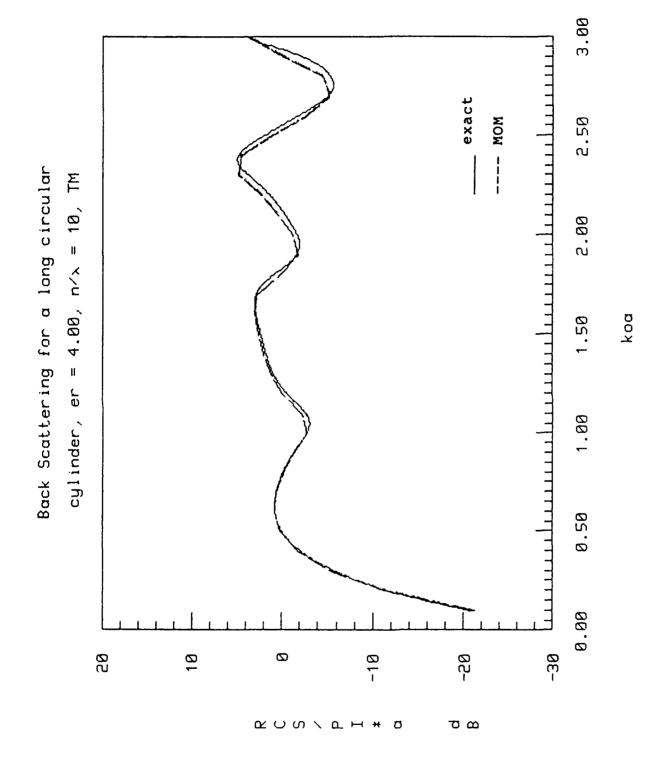
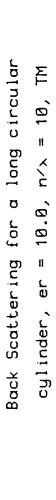


Fig. 45



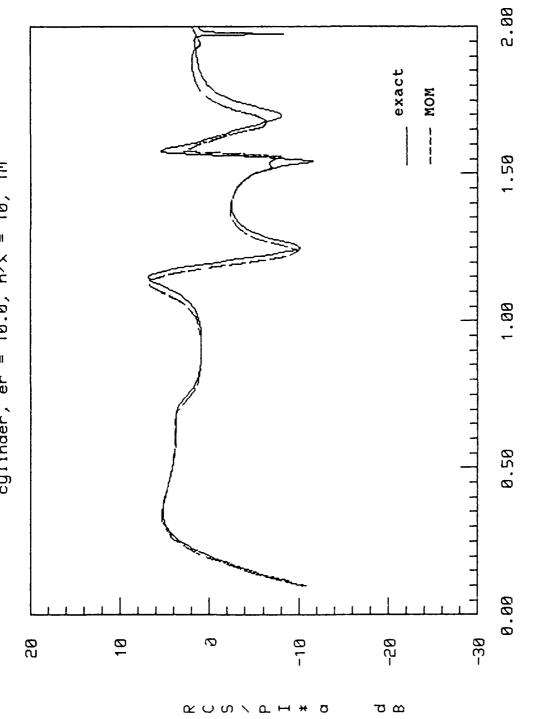


Fig. 46

koa

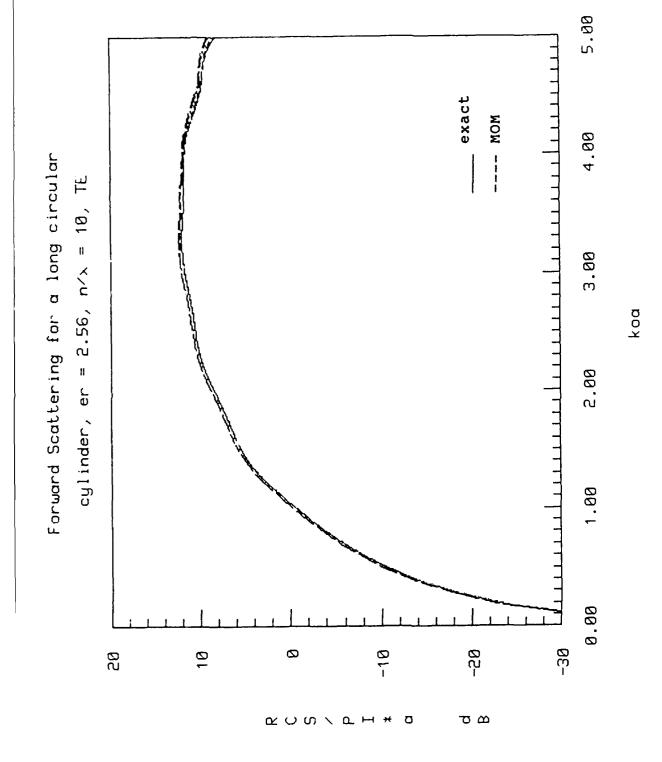


Fig. 47

3.00 exact 2.50 MOM ----Forward Scattering for a long circular cylinder, er = 4.80, n/x = 10, TE 2.00 1.50 1.88 0.50 0.00 -30 20 10 -20 -10 $\boldsymbol{\omega}$ BB αυσ \ a H *

Fig. 48

koa

3.88 exact ---- MOM 2.50 Back Scattering for a long circular cylinder, er = 4.00, n/x = 10, TE 2.00 κοα 1.50 1.88 0.50 00.00 -30 28 10 Ø -10 -20 $\alpha \circ \alpha \vee \alpha$ **в** 0

Fig. 49

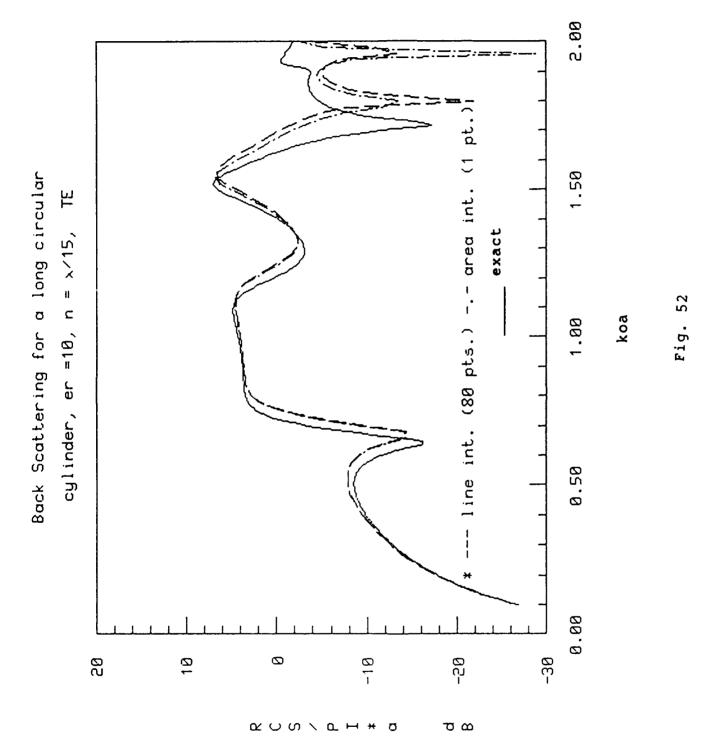
5.00 exact ---- MOM 4.00 Back Scattering for a long circular cylinder, er = 2.56, n/λ = 10, TE 3.00 κοα 2.00 1.00 0.00 -30 28 18 Ø -10 -20 B B

Fig. 50

Figs. 47-50 show the forward and back scattering from the dielectric circular cylinder versus k_oa with relative permittivities of 2.56 and 4 respectively for TE polarization. Good agreement exists between the different methods (120 k_oa points) and the exact series solution (1200 k_oa points). These is no significant distinction between the exact solution and the three methods when the relative permittivity is small.

However, as the permittivity is increased, the computed solution deviates from the exact solution. This is especially true for the TE case when the relative permittivity is increased to 10. Shown in Fig. 51 is the forward scattering from a long lossless circular cylinder versus koa with two different methods, the single center point area integral and the line integral method. The number of segments per dielectric wavelength is set to 15. The curves begin to deviate from the exact solution (the solid curve) when $k_0a > 0.5$. Fig. 52 shows the back scattering. Even though, the two numerical methods produce curves that differ somewhat, the line integral method for evaluating the self cell terms deviates less than the center point area integral method. It is noted

2.00 Forward Scattering for a long circular --- line int. (80 pts.) -.- area int. (1 1.50 TE cylinder, er =10, n = \times 715, 1.00 koa exact 0.50 0.00 28 -30 10 Ø -10 -20 **8** 0 & O S / A H *



2.00 Forward Scattering for a long circular 1.50 Line integral method with 80 points. cylinder, er =10, n = x/10,15,18; 1.00 0.50 exact 20 Ø -20 & O S / P H * ထတ

2.88 끧 1.58 Back Scattering for a long circular Line integral method with 80 points. cylinder, er =10, n = x/10,15,18; koa 1.88 0.50 exact 0.00 -30 -18 -20 20 0 O 8 U S / B H *

Fig. 54

that a sharp drop occurs at $k_o a = 1.95$ in the solution with center point area integral method that is not found in the exact solution or with the line integral.

Figs. 53-54 shows a slight improvement when the number of cells per dielectric wavelength is increased. However, the curves seem to converge more slowly. The closest match is the curve where the number of cells per dielectric wavelength is set to 18. The greatest mismatch is found when it is set to 10. Due to the limitation of computer time and storage, the maximum size k_0a of the cylinder is set to 1.8 for n/x = 18. This gave an 1800×1800 matrix.

From Figs. 53 and 54, we select three points to observe the scattering from the dielectric circular cylinder versus scattering angles. The three points are $k_0a = 0.5$ (good agreement with exact solution), 1.5 (slight disagreement with exact solution) and 1.7 (poor agreement with exact solution). Figs. 55-57 show the normalized scattering from a lossless dielectric circular cylinder when $\epsilon r = 10$, and $k_0a = 0.5, 1.5, 1.7$ respectively. The number of segments per dielectric wavelength is set to 10. As seen from figures, the curves of scattering versus angle confirm the agreement at the single

180 exact 160 MOM ----148 circular cylinder, er =10, koa=0.5, TE Normalized Scattering for a long 120 100 80 9 40 28 0 -40 Ø -20 10 -18 -30

 α \circ \circ \circ \circ \circ \circ

Fig. 55

angle (degrees)

8 0

180 160 exact ---- MOM 140 circular cylinder, er =10, koa=1.5, TE Normalized Scattering for a long 120 angle (degrees) 100 80 9 40 20 Ø -30 20 10 Ø -18 -20 മെ & O S / P H *

Fig. 56

Normalized Scattering for a long circular cylinder, er =10, koa=1.7, TE

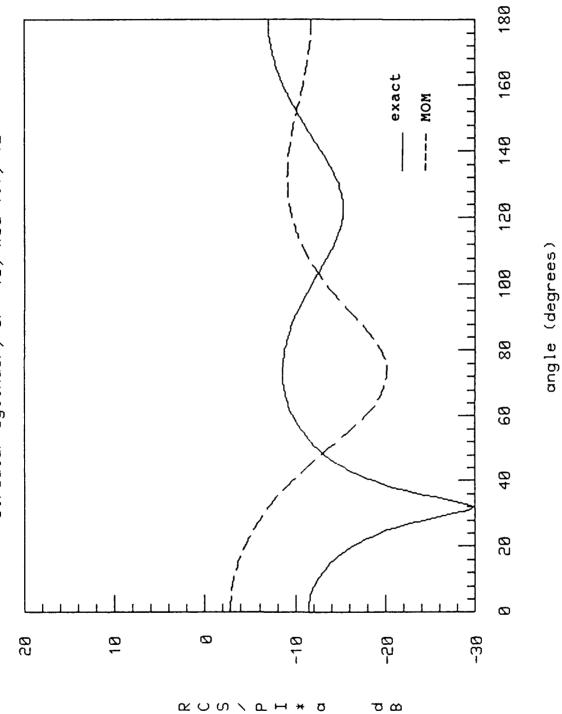


Fig. 57

6.00 5.88 Forward Scattering for a long square cylinder, er =2.56, n = x/10, 4.00 3.00 коа 2.88 1.38 8.88 -38 20 1 13 Ø - 10 -20 & O W / Ø H H σ σ

4.00 Forward Scattering for a long square 3.00 Σ cylinder, er =4.88, n = x/18, k oa 2.00 1.88 8.88 -38 Ø <u>6</u> -18 20 -28 ασ & O O V O F H # 0

78

3.88 Forward Scattering for a long square Ε 2.88 = x/18, cylinder, er =10.0, n 1.88 0.00 -30 20 <u>6</u> Ø -16 128 αυνναμ * ο α σ

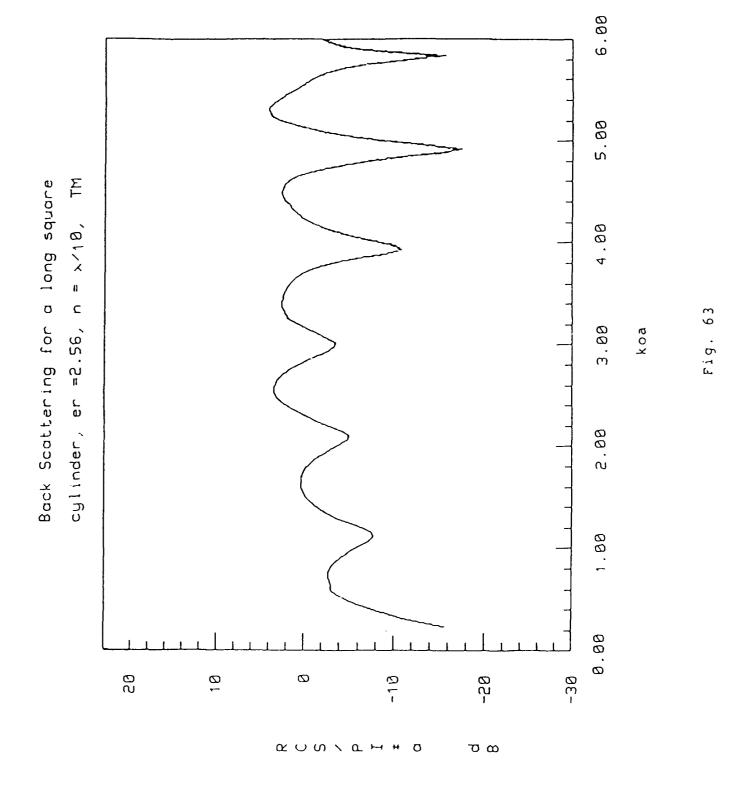
koa

79

6.00 5.88 Forward Scattering for a long square 띰 4.00 = ×/18, C 3.00 cylinder, er =2.56, koa 2.00 1.00 0.00 -18 -30 28 Ø -20 18 ασ αυσ×αн* Q

4.88 Forward Scattering for a long square 3.00 띰 cylinder, er =4.00, n = x/10, 2.88 koa 1.88 8.88 -30 20 18 Ø -18 -28 **σ** σ & C C V V P H * D

81



4.00 3.88 ٤ Back Scattering for a long square cylinder, er =4.88, n = x/18, koa 2.88 1.88 8.88 -38 28 18 ß -20 മെ αυσ×α H * Ø

83

3.88 Ε Back Scattering for a long square 2.88 = x/18, C Fig. 65 koa cylinder, er =10.0, 1.88 9.88 -38 <u>5</u> Ø S B -10 -20 αυσ×α H * α σ

8 4

Back Scattering for a long square 띰 $= \times 10$ cylinder, er =2.56, n

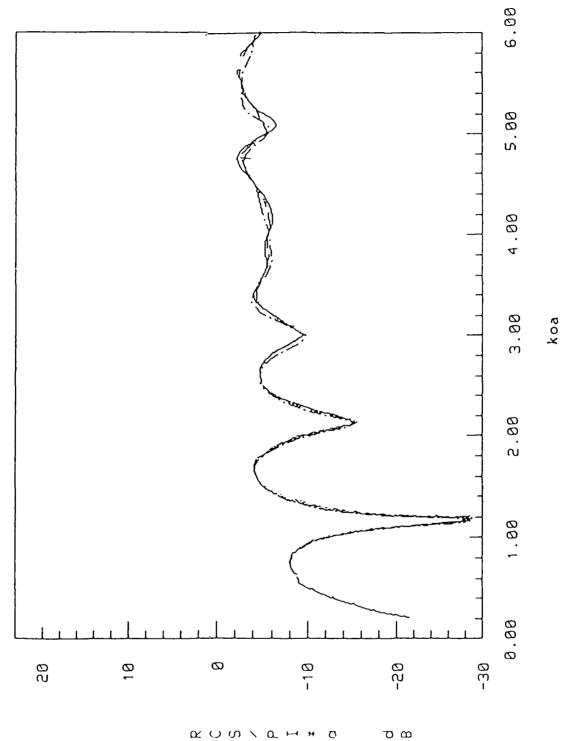


Fig. 66

4.88 3.00 Back Scattering for a long square 띰 cylinder, er =4.88, n = x/18, 2.00 1.88 0.00 20 Ø -38 10 -18 -20 & O & X & H * σœ

Fig. 67

koa

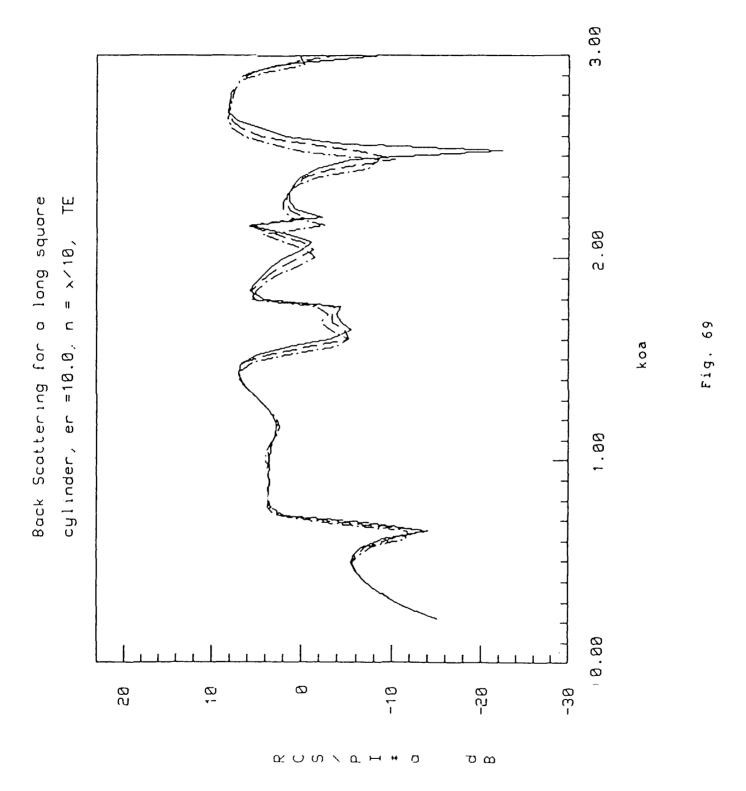
points. That is, Fig. 55 shows a close match to the exact solution, and Fig. 56 shows a slight mismatch from the exact solution, and Fig. 57 shows a complete mismatch.

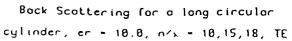
The next set of figures (Figs. 58-62) represent the forward scattering from a long square dielectric lossless cylinder, with relative permittivities of 2.56,4, and 10 for TM polarization, and 2.56 and 4 for TE polarization. The number of cells per dielectric wavelength is set to 10 to ensure more accuracy. As before, three methods of integrating the Green's function over the cells are used. The back scattering is shown in Figs. 63-67. As seen from the figures, there is no significant difference between the results of the three methods.

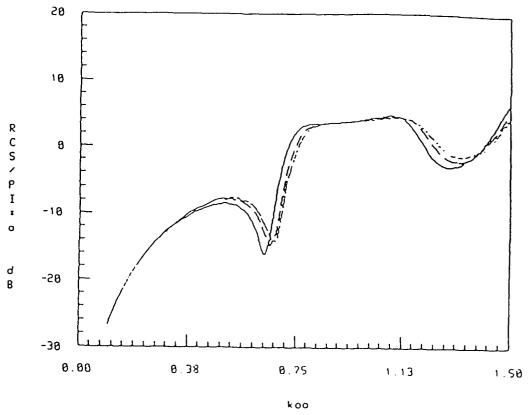
Figs. 68-69 show the TE forward and back scattering from a dielectric square cylinder with relative permittivity of 10 for the three different methods. The number of cells per dielectric wavelength is set to 10. The total number of k, a points used is 100. As seen from these curves, the three methods give results that vary slightly from each other. The solid curve represents the line integral approximation of the cells, the single dotted line curve represents the

3.00 Forward Scattering for a long square Щ 2.88 n = x/10, Fig. 68 =10.0, кoа n L cylinder, 1.88 9.88 28 6 Ø -30 -20 αυν \ α H * α σœ

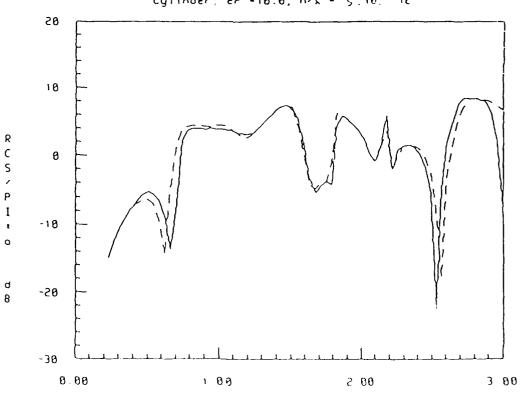
88







Bock Scottering for a long square cylinder, or =10.0, n/x = 5.10. IE



koa

area integral method with one center point. As more points are added to the area integration over the cells (e.g., 25 points in our case), the curve tends to converge to the solid curve represented by the line integral method which has 80 points.

Fig. 70 shows the back scattering from a circular cylinder and a square cylinder using three different cell sizes for TE polarization with $\epsilon r = 10$. It can be seen that decreasing the cell size tends to produce convergence to the exact solution (solid curve) for the circular cylinder. However, the curves converges very slowly. For the square cylinder, notice that at $k_0a = 3.0$ the curves for two different cell sizes seem to diverge even though the square boundary matches perfectly, and there is no jagged edges as in the case of the circular cylinder. This confirms that the jagged edges of the circular cylinder are not a large source of error.

6. CONCLUSION

The special treatment of the self cells in the volume integral equation did improve the performance of the solution for scattering from dielectric bodies. However, the improvement was slight and limited to small scattering bodies. When the relative permittivity is set equal to 10, the total number of cells needed for TE polarization is already in the range of 1800 for k_oa=1.8. Thus the size of the scatterer we can handle on our mainframe computer is limited. Moreover, the curves from Figs. 68-69 require at least 120 k_oa points. It is very time consuming to run the simulation program 120 times to get the required curve.

One source of the inaccuracies of the solution is the square-cell approximation of the cross section of the circular cylinder. However, our computations indicated that this jagged edges problem is not a major source of error.

The most important reason for the inaccuracies, as pointed out by Peterson [1], is in the pulse-basis, point-matching formulation. Using the pulse basis functions introduces fictitious charge layers at every cell boundary with resulting

numerical error. This error tends to increase as the relative permittivity increases. However, with a new formulation of the volume integral equation which deals with the surface charge density, we hope to overcome these inaccuracies and obtain a benchmark solution for multi-wavelength dielectric scatterers using convenient pulse basis functions. Finally, making use of the symmetry of the matrices and converting to Toeplitz or tridiagonal matrices will ensure faster computing time and less storage requirements.

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